Implicit surface tension model for stimulation of interfacial flows

Vinh The Nguyen

University of Massachusetts Dartmouth

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About my project

Project Advisor

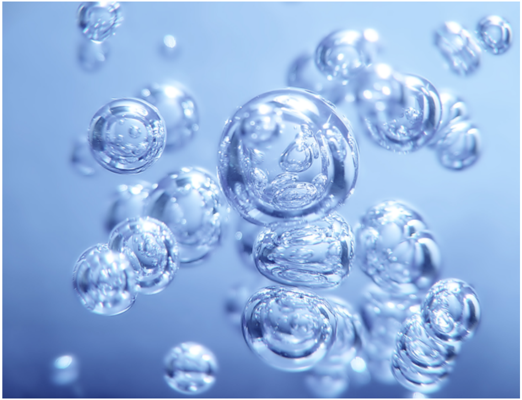
- Dr. Mehdi Raessi
- Department of Mechanical Engineering

Project Objective

- To study implicit modeling of surface tension.
- To generate faster and better model that produce no spurious currents and has larger time step restriction.

Introduction

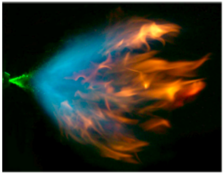
- What is interfacial flow?



- What are the applications?

Interfacial flows, important in many applications

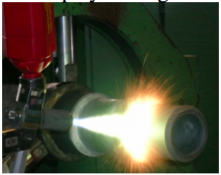
Combustion



Courtesy of Edwards

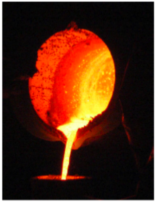
Naval hydrodynamics

Spray coating



Spray cooling

Casting



Modeling of interfacial flow

- To accurately model the interfacial flow is challenging because:
- The discontinuity of fluid properties (such as density).
- The interfacial boundary condition (surface tension).

Currently there are two main interfacial flow models

- Explicit:
  Precise but has high computational cost due to small time step restriction.
- Implicit:
  Lower computational cost than explicit due to higher time-step restriction.
  Draw backs: appearing of nonphysical velocities (spurious current)
  Common flow solver: Continuum surface force (CSF)

Method used

CSF method

- The implicit model studied in this project is based on Continuum Surface Force method
- This method was first proposed by Brackbill et al. 1991

Drawbacks:

- This method generates unphysical velocities (spurious currents)
- The spurious current is caused by:
- Imbalance of the surface tension and pressure gradient.
- Error in computing the curvature. (this project)

$$\vec{F} = (\sigma\kappa\hat{n} + \nabla_s\sigma)\delta$$

(Francois et al. 2007)
The pressure drop across the interface:

$$\Delta p = p_2 - p_1 = \sigma k \tag{1}$$

σ is the surface tension coefficient
k is the mean curvature

$$k = \frac{1}{R_I} + \frac{1}{R_{II}} \tag{2}$$

δ is the delta function represent interface (Raessi et al. 2008)

My task

Tasks:

- Study the CSF model
- Study the stability of the model (CFL condition) as time-step increases using different curvature solving method: Level set (LS) and Advecting Normal
- Compare the results with exact curvature.

Challenges:

- To get used to the code and understand what's the function of each parts requires lots of trials errors.
- To manipulate it to do what I want also requires lots of trials and errors.

CFL number/ CFL condition

- The Courant-Friedrichs-Lewy condition (CFL condition) is a necessary condition for convergence while solving partial differential equations numerically.

$$CFL = \frac{u.\Delta t}{\Delta x} \leq C \tag{3}$$

C= 1

Level Set method

In the LS method, the interface is represented by a smooth function ϕ – called the LS function; for a domain Ω, ϕ is defined [15] as a signed distance to the boundary (interface) $\partial\Omega$

$$|\phi(\vec{x})| = \min(|\vec{x} - \vec{x}_I|) \quad \text{for all } \vec{x}_I \in \partial\Omega \tag{6}$$

implying that $\phi(\vec{x}) = 0$ on $\partial\Omega$. Choosing ϕ to be positive inside Ω, we then have

$$\phi(\vec{x}) = \begin{cases} > 0, & \vec{x} \in \Omega \\ 0, & \vec{x} \in \partial\Omega \\ < 0, & \vec{x} \notin \Omega \end{cases} \tag{7}$$

For the 2D interface depicted in Fig. 1a, the discretized LS function, defined at the center of each cell, is shown in Fig. 1c.

The unit normal vector and curvature at any point on the interface are calculated from ϕ by

$$\hat{n} = \frac{\nabla\phi}{|\nabla\phi|} \tag{8}$$

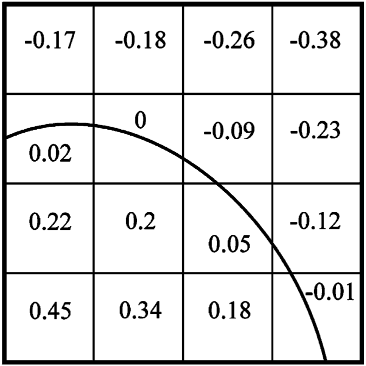
and

$$\kappa = -\nabla \cdot \left(\frac{\nabla\phi}{|\nabla\phi|}\right) \tag{9}$$

(Raessi et al. 2007)

Level Set method

c



(Raessi et al. 2007)

Advecting Normal method

As reviewed earlier, the evolution of the LS function is governed by Eq. (10)

$$\frac{\partial \phi}{\partial t} + \vec{u} \cdot \nabla \phi = 0$$

Defining $\vec{N} = \nabla \phi$ as the vector normal to the contours of ϕ, the above equation can be rewritten as

$$\frac{\partial \phi}{\partial t} + \vec{u} \cdot \vec{N} = 0$$

Taking the gradient of Eq. (14), we obtain

$$\frac{\partial \vec{N}}{\partial t} + \nabla \left(\vec{u} \cdot \vec{N} \right) = 0 \tag{15}$$

Eq. (15) is the advection equation for normals. In 2D Cartesian coordinates, Eq. (15) results in the following equations:

$$\frac{\partial N_x}{\partial t} + \frac{\partial}{\partial x}(uN_x + vN_y) = 0 \tag{16}$$

and

$$\frac{\partial N_y}{\partial t} + \frac{\partial}{\partial y}(uN_x + vN_y) = 0 \tag{17}$$

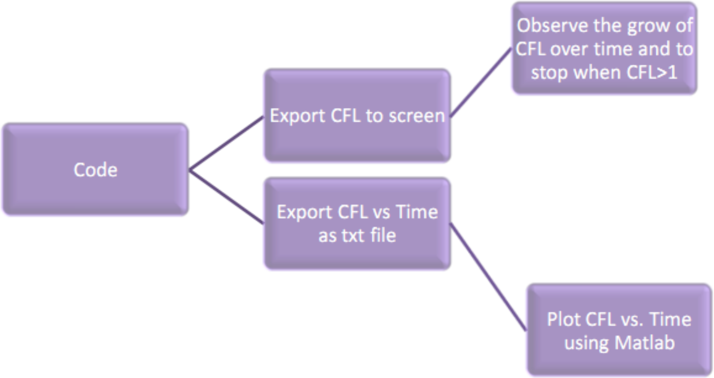
(Raessi et al. 2007)

Case study: Static drop

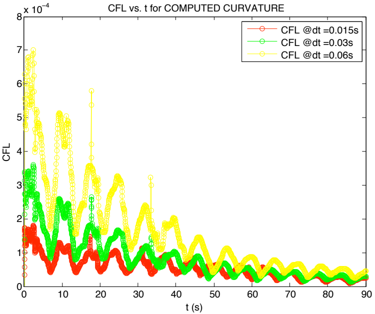
- Static water drop in zero gravity.
- $\rho 1 = \rho 2 = 10^3 Kg/m^3$
- $\mu 1 = \mu 2 = 0.05$
- $g = 0$
- Surface tension time-step restriction $\Delta t_{ST} = 0.03$

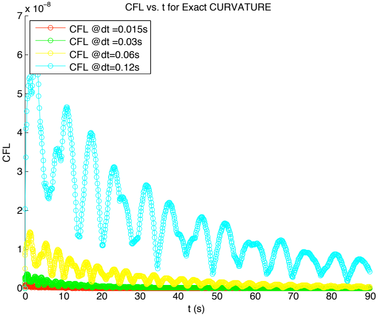
(Raessi et al. 2008)

What I did


Code
Export CFL to screen
Export CFL vs Time as txt file
Observe the grow of CFL over time and to stop when CFL>1
Plot CFL vs. Time using Matlab

Results - Level Set method vs. Exact Curvature





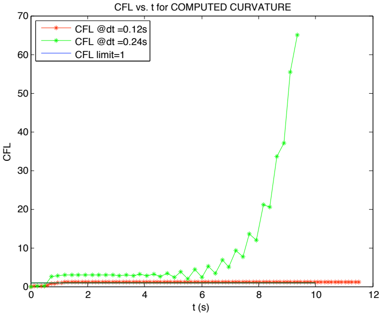
- Level set
- Exact curvature

The timestep was increase as : $\Delta t = 0.5, 2, 4\Delta t_{ST}$

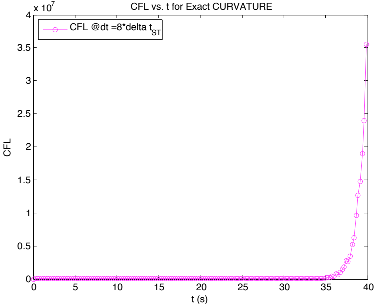
Result- Level set at $dt = 4, 8dt_{ST}$

Implicit modeling

Nguyen

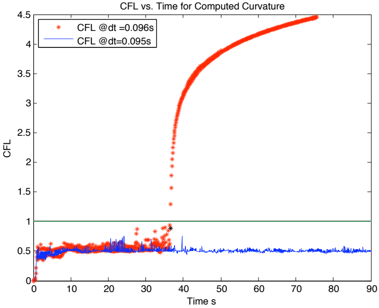


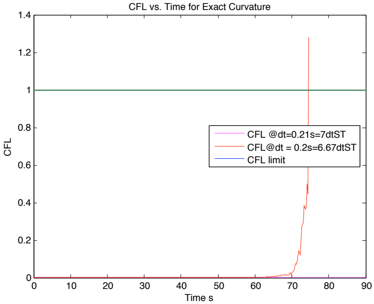
- Level set $dt = 4, 8\Delta t_{ST}$



- Exact curvature $dt = 8\Delta t_{ST}$

Result- Level set vs. Exact Curvature for maximum timestep





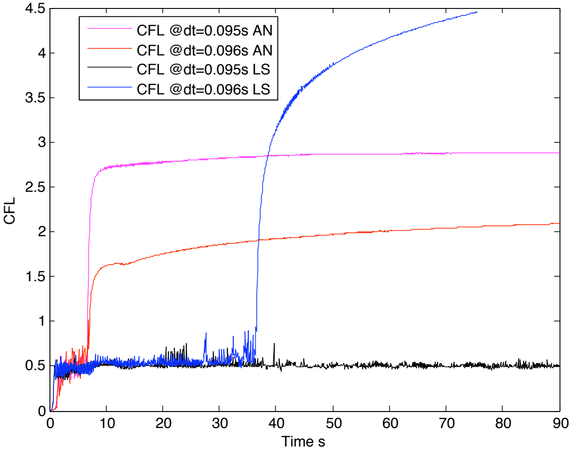
- Level set
  $dt = 3.167, 3.2\Delta t_{ST}$

- Exact curvature
  $dt = 6.67, 7\Delta t_{ST}$

Result- Level set vs Advecting Normal

Implicit modeling

Nguyen



Question

The errors associated with curvatures calculated from the LS function ϕ, for a circle of radius 0.15 centered at (0.5,0.5) in a 1×1 domain, at different mesh resolutions

Δx	l_∞	Order	l_1	Order
1/16	0.5472		0.2963	
		1.55		1.58
1/32	0.1875		0.0991	
		−1.79		−0.51
1/64	0.6481		0.1407	
		0.61		−0.11
1/128	0.4234		0.1518	
		−0.43		−0.02
1/256	0.5689		0.1537	
		−0.29		0.34
1/512	0.6963		0.1215	
		0.11		−0.01
1/1024	0.6453		0.1227	

Table 6
The errors associated with curvatures calculated by the $\vec{N}$ method, for a circle of radius 0.15 centered at (0.5,0.5) in a 1×1 domain, at different mesh resolutions

Δx	l_∞	Order	l_1	Order
1/16	3.87×10^{-1}		2.11×10^{-2}	
		1.72		1.73
1/32	1.18×10^{-1}		6.33×10^{-2}	
		2.11		2.20
1/64	2.73×10^{-2}		1.38×10^{-2}	
		2.07		1.94
1/128	6.51×10^{-3}		3.60×10^{-3}	
		1.95		2.03
1/256	1.68×10^{-3}		8.83×10^{-4}	

Conclusion

Proved:

- To accurately compute the curvature is crucial and it can increase the stability of the solution for implicit model

Future Research

- Continue to study the implicit models.
- Starting with the simulation.

References

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