Chaotic Mixing In a Shear Flow Environment

Daniel Kelleher and Amit Tandon
University Of Massachusetts Dartmouth Physics Department
Contact Email: dkelleher@umassd.edu

Background

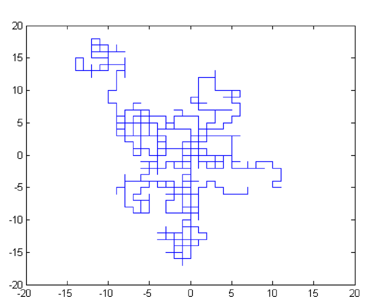
The purpose of this research was to explore chaotic mixing without the use of the advection-diffusion equation

$$\frac{\partial \phi}{\partial t} + v \cdot \nabla \phi = k \nabla^2 \phi \qquad (1)$$

To achieve this a random walk was used to simulate the mixing and a velocity field was introduced to effectively stir the fluid.

Random Walk

Random walk is a statistical algorithm, here taken with a constant step size where the point at one vertex may move to any directly adjacent vertex. The resulting probability density function mimics a gaussian curve and have an root mean square pathlength of $x_{rms} \propto \sqrt{t}$ where t is the number of time steps.



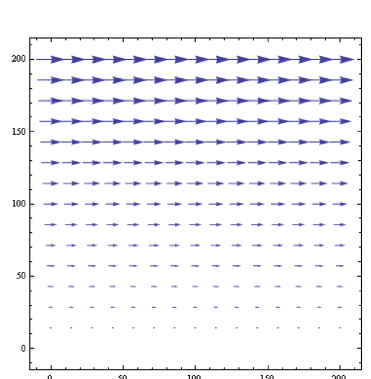
The above example is 5 particles evolved over 150 time steps. While a gaussian curve is not easy to see from this image it is clear that the particle has a much higher probability to stay around the origin.

Shear Velocity

An incompressible shear velocity field is usually defined by

$$\nabla \cdot v = 0 \qquad (3)$$

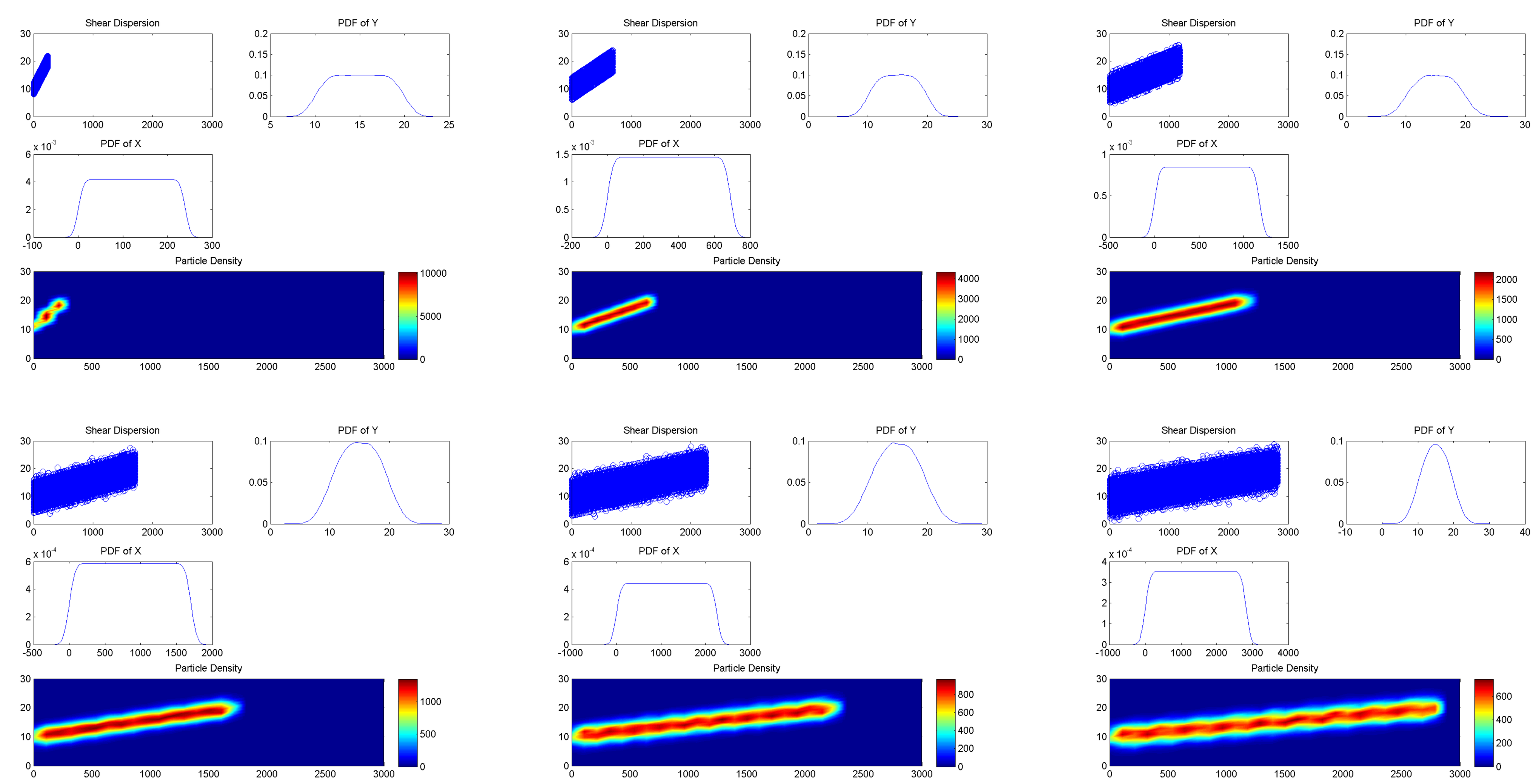
$$\frac{dU}{dy} = C \qquad (4)$$



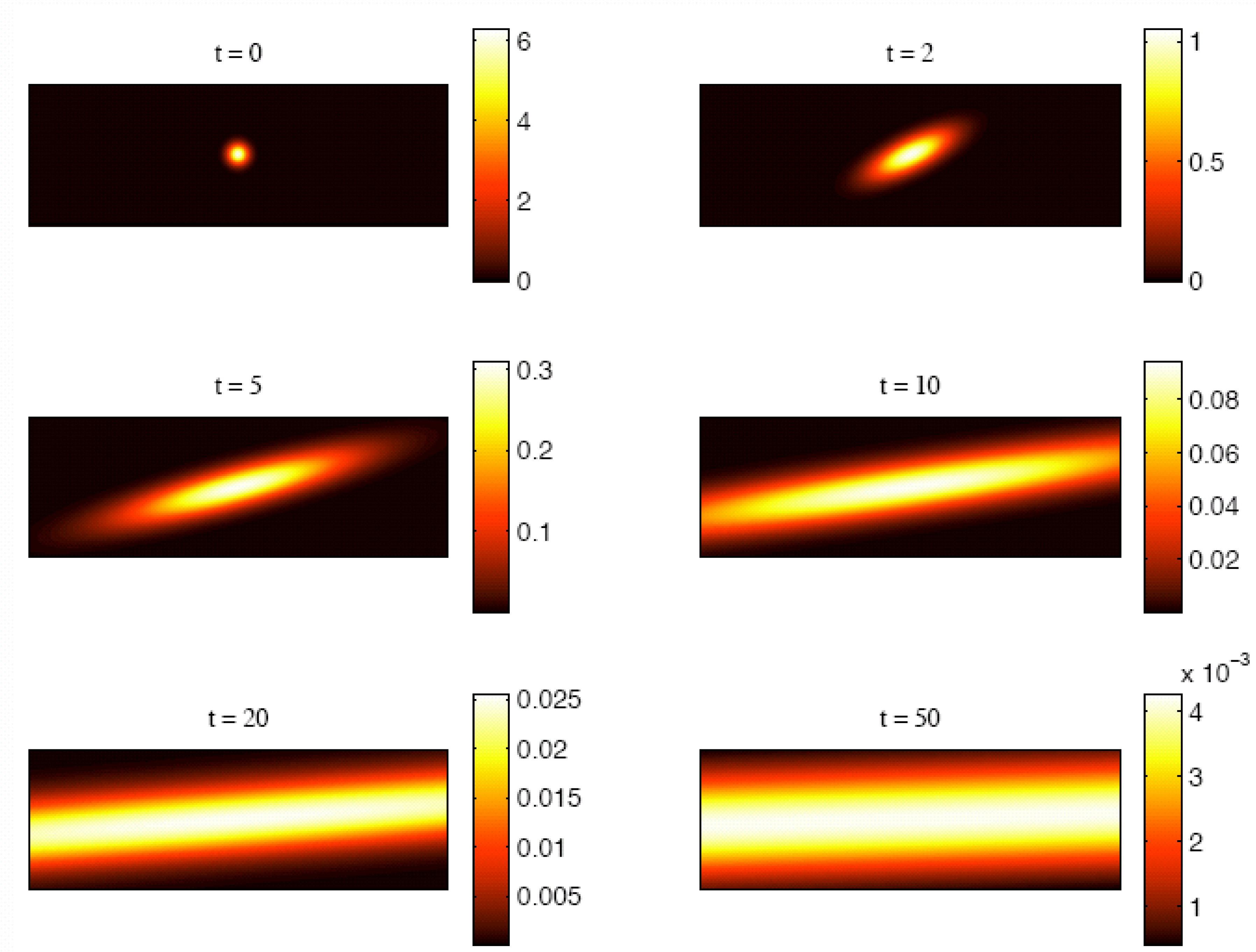
Where C is a constant. Shear fields are important in the study of fluid dynamics because they appear in many forms of chaotic turbulence. An example of the shear flow used in this experiment is shown above.

Results

The sequence of images in this panel shows the locations of 1 million particles evolved over 12 random walk time steps in one of the shear dispersion experiments, where random walk is superposed on the shear flow (Young *et al*. 1982). In this experiment, the random walk time scale is longer than the shear time scale by a factor of 3. The snap shots are taken at t= 2, 4, 6, 8, 10, and 12 respectively.



The results from this computational experiment were compared to the results from a similar experiment using the exact advection-diffusion equation (equation 1) from Scalar Decay in Chaotic Mixing (Thiffeault 2008).



It is clear that although the experiments where done in different reference frames relative to the diffusion they are almost identical.

Acknowledgement

This research was supported by the national science foundation CSUMS program and the University of Massachusetts Dartmouth Math Department. Finally this research was made possible by the continued support of Dr. Amit Tandon.

FTLEs

One form of analysis which we tried was finite time Lyapunov exponents(FTLE).

$$\sigma_{t_0}^{T} = \frac{1}{|T|} ln \sqrt{\lambda_{max}(\Delta)} \qquad (2)$$

We hope to use this in the future.

Open Questions

These are questions we hope to answer by the end of this semester.

- How can this model be used to simplify our understanding of real world situations.
- What are the computational demands of this model for large scale velocity fields which evolve with time.
- How applicable is this model for different PDF curves for random walk (e.g. Levy Flights, or Gaussian distribution)

Future Work

We hope to work on the following before graduation next year.

- Model the tracer distribution around an oceanic eddy with this method
- Model an FTLE system using this method and compare results

References

J.-L. Thiffeault, Scalar Decay in Chaotic Mixing, in *Transport and Mixing in Geophysical Flows*, Lecture Notes in Physics **744**, 3-35, 2008 (Springer, Berlin). (Proceedings of the Gran Combin Summer School, Valle d'Aosta, Italy, 14-24 June 2004.)

Young, W.R, Rhines, P. and Garrett C., Shear-Flow Dispersion, Internal Waves and Horizontal Mixing in the Ocean, *Journal of Physical Oceanography*, **12**, 515-527, 1982.